[2]Sharf, I., D'Eleuterio, G. M. T., and Hughes, P. C., "On the Dynamics of Gibbs, Appell, and Kane," *European Journal of Mechanics. A, Solids*, Vol. 11, No. 2, 1992, pp. 145–155.

[3]Kane, T., "Impulsive Motions," *Journal of Applied Mechanics*, Vol. 29, No. 4, 1962, pp. 715–718.

[4]Kane, T. R., and Levinson, D. A., *Dynamics: Theory and Applications*, 1st ed., McGraw–Hill Series in Mechanical Engineering, McGraw–Hill, New York, 1985.

[5]Goldsmith, W., *Impact*, Edward Arnold, London, 1960.

[6]Haug, E., Wehage, R., and Barman, N., "Design Sensitivity Analysis of Planar Mechanism and Machine Dynamics," *Journal of Mechanical Design*, Vol. 103, No. 3, 1981, pp. 560–570.

[7]Lankarani, H., and Nikaravesh, P., "Canonical Impulse–Momentum Equations for Impact Analysis of Multibody Systems," *Journal of Mechanical Design*, Vol. 114, No. 1, 1992, pp. 180–186.

[8]Khulief, Y., and Shabana, A., "Dynamic Analysis of Constrained Systems of Rigid and Flexible Bodies with Intermittent Motion," *Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 108, No. 1, 1986, pp. 38–44.

[9]Chang, C.-C., Liu, C., and Huston, R., "Dynamics of Multibody Systems Subjected to Impulsive Constraints," *Multibody System Dynamics*, Vol. 8, No. 2, 2002, pp. 161–184.

[10]Bajodah, A. H., Hodges, D. H., and Chen, Y. H., "New Form of Kane's Equations of Motion for Constrained Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 1, 2003, pp. 79–88.

Differentiator-Free Nonlinear Proportional-Integral Controllers for Rigid-Body Attitude Stabilization

Kamesh Subbarao*
The University of Texas at Arlington, Arlington, Texas 76019
and
Maruthi R. Akella†
The University of Texas at Austin, Austin, Texas 78712

I. Introduction

THE past two decades have witnessed several important developments in the design of feedback controllers for rigid-body attitude stabilization (set-point regulation) and maneuver tracking objectives.[1,2] However, there still remain certain open problems in this field that are of great theoretical and practical interest. In particular, from the standpoint of external disturbance torque rejection, there currently exists no unified framework for designing simple control structures inspired from linear control theory such as proportional-integral (PI), proportional-integral-derivative (PID), and their variants. The main hindrance, of course, stems from the fact that the governing differential equations for the kinematics and dynamics of rigid-body attitude motion are nonlinear in nature.

In this Note, we present a novel framework of constructing PI-like controllers for attitude stabilization without using body angular rate (velocity) measurements. Our interest in the velocity-free control framework is largely motivated from practical considerations wherein the assumption of availability of the angular velocity measurement is not always satisfied because of either cost limitations or implementation constraints (e.g., tachometers are not available with all robotic manipulators). The earliest known result in the field of velocity-free attitude control was given by Lizarralde and Wen[3] through the passivity framework. This result utilizes the Euler parameter (quaternion) kinematics and a passivity filter whose state enables the construction of the attitude stabilizing controller. The structure of the filter vector state is shown to be governed through a stable first-order linear differential equation driven by the vector part of the quaternion (attitude measurement). Interestingly, the passivity filter does not have high-pass characteristics and in theory admits arbitrarily slow bandwidth. Also, for sufficiently small frequencies in the filter's input signal, the filter output approximates a pseudo-velocity-like state. However, this is not true for input signals with higher magnitudes of frequencies.

Subsequent extensions to this velocity-free controller framework were presented by Tsiotras[4] for kinematics expressed in terms of the nonredundant Gibbs vector and the modified Rodrigues parameters vector sets. Further generalizations to the case of attitude tracking along prescribed reference trajectories were given by Akella[5] and Akella and Kotamraju.[6]

One important merit possessed by all of these passivity-based schemes is that the underlying velocity-free control law is independent of the inertia matrix for the case of set-point regulation, thereby automatically providing stability robustness in the presence of arbitrarily large inertia parameter errors. At the same time, none of the aforementioned results provide room for integral control action that can potentially help eliminate or reduce steady-state attitude error in the presence of constant external disturbance torques. In a recent development, Subbarao[7] approached the attitude stabilization problem from the nonlinear PID control perspective wherein the full-state vector including the angular velocity was employed for feedback purposes. Even though the control scheme requires full-state feedback when compared to the passivity-filter based constructions, this result brings the useful provision for integral feedback. In terms of robustness with respect to inertia parameter uncertainties, the PID construction of Subbarao[7] is nearly independent of the inertia matrix in the sense that it implicitly requires only prior knowledge on the largest eigenvalue of the inertia matrix. Attitude control with globally stable closed-loop dynamics employing the Euler parameters was also discussed in Ref. 8. The main result in that paper was derived from a feedback-linearization-like approach, wherein linear attitude error dynamics was prescribed and the control law was derived to enforce the linear error dynamics. The feedback control law assumed the knowledge of the attitude quaternion and the angular velocity information for implementation.

The fundamental contribution of this Note is the derivation of a new class of PI controllers for the attitude stabilization problem without using explicit velocity feedback. This work combines aspects of the passivity filter formulations together with the choice of a Lyapunov function containing cross/mixed terms involving the various states. The requirement for such cross terms is dictated by the presence of integral feedback terms within the control law that cannot otherwise be accommodated within the conventional passivity filters. As will be illustrated through the subsequent discussions, the important consequence is that we are able to guarantee global asymptotic stability for the nonlinear closed-loop dynamics in the ideal case (zero external disturbances) without requiring any sort of prior information on the body inertia matrix. Further, through numerical simulations we will evaluate potential advantages derived through the inclusion of the integral feedback term within the control law by computing the attitude error convergence in the presence of unknown (constant) disturbance torques. Local disturbance rejection is illustrated through a linearization about the nominal equilibrium point.

Presented as Paper 2004-202 at the AAS/AIAA 14th Space Flight Mechanics Meeting, Maui, Hawaii, 8–12 February 2004; received 17 February 2004; revision received 1 June 2004; accepted for publication 1 June 2004.

*Assistant Professor, Department of Mechanical Aerospace Engineering; subbarao@mae.uta.edu. Member AIAA.

†Assistant Professor, Department of Aerospace Engineering and Engineering Mechanics; makella@mail.utexas.edu. Member AIAA.

The paper is organized as follows: In Sec. II, we state the governing nonlinear dynamics concerning rigid-body rotational motion and the control objective for attitude stabilization. The details of PI controller structure are stated in Sec. III together with the necessary Lyapunov stability analysis. The disturbance rejection feature is also discussed qualitatively. Numerical simulation results are presented in Sec. IV, followed by concluding remarks in Sec. V.

II. Attitude Dynamics and Statement of the Control Objective

The rotational motion of a rigid body is described by the well-known Euler's equation

$$J\dot{\omega} = -S(\omega)J\omega + \boldsymbol{u} + \boldsymbol{d} \tag{1}$$

where $\omega(t) \in \mathcal{R}^3$ is the angular velocity of rigid body given in a body-fixed frame of reference, $J = J^T \in \mathcal{R}^{3\times 3}$ is the positive-definite mass moment of inertia matrix, and $\boldsymbol{u}(t) \in \mathcal{R}^3$ is an external torque input vector. The bounded unknown external disturbance torque is defined through $\boldsymbol{d}(t) \in \mathcal{R}^3$. Also in Eq. (1), $S(\cdot)$ denotes the skew-symmetric matrix operator that performs the vector cross product in such a way that $S(\boldsymbol{a})\boldsymbol{b} = \boldsymbol{a} \times \boldsymbol{b}$ for every $\boldsymbol{a}, \boldsymbol{b} \in \mathcal{R}^3$.

As far as the attitude kinematics are concerned, we adopt the unit quaternion vector $\boldsymbol{\beta}(t) \in \mathcal{R}^4$ to designate the attitude of the rigid body relative to any target attitude (setpoint). The unit quaternion representation of the direction cosine (rotation) matrix is defined by the following:

$$\boldsymbol{\beta} = [\beta_0, \beta_1, \beta_2, \beta_3]^T = [\beta_0, \boldsymbol{\beta}_v]^T \tag{2}$$

Further details about the quaternion, its usage in description of attitude kinematics, and several associated properties can be found in Refs. 9 and 10. We summarize the kinematic differential equations as

$$\dot{\boldsymbol{\beta}} = \tfrac{1}{2} E(\boldsymbol{\beta})\boldsymbol{\omega} \tag{3}$$

where the matrix $E = E(\boldsymbol{\beta})$ is given by[3,9]

$$E(\boldsymbol{\beta}) = \begin{bmatrix} -\boldsymbol{\beta}_v^T \\ \beta_0 I_3 - S(\boldsymbol{\beta}_v) \end{bmatrix} \tag{4}$$

with I_3 being the 3×3 identity matrix. It can be easily verified from the preceding definition that

$$E^T(\boldsymbol{\beta})E(\boldsymbol{\beta}) = I_{3\times 3}, \qquad E^T(\boldsymbol{\beta})\boldsymbol{\beta} = 0 \tag{5}$$

Consequent to Eq. (3), the angular velocity vector can be expressed in terms of $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$ as

$$\boldsymbol{\omega} = 2E^T(\boldsymbol{\beta})\dot{\boldsymbol{\beta}} \tag{6}$$

Thus, we adopt Eqs. (1) and (3) to govern the nonlinear dynamics of rigid-body rotational motion over the seven dimensional $[\boldsymbol{\beta}, \boldsymbol{\omega}]^T$ space subject to the unit norm constraint on $\boldsymbol{\beta}$.

The control objective can now be stated as follows: For the ideal case when external disturbances are completely absent $[\boldsymbol{d}(t) = 0]$, the goal is to determine a PI type of input torque vector $\boldsymbol{u}$, which is independent of angular velocity $\omega(t)$ measurement/feedback in such a way that the closed-loop trajectories described by Eqs. (3) and (1) are globally stable and they converge to the set

$$\Omega = \{\beta_0 = \pm 1, \boldsymbol{\beta}_v = 0, \boldsymbol{\omega} = 0\}$$

III. Nonlinear PI Control Design

In this section, we show that when $\boldsymbol{d}(t) = 0$ for all $t \geq 0$ the states $\boldsymbol{\beta}$ and $\boldsymbol{\omega}$ governed by Eqs. (3) and (1) can be controlled through a PI framework without angular velocity $\boldsymbol{\omega}$, feedback, and thus, only orientation $\boldsymbol{\beta}$ feedback is required. In the following theorem, we summarize our control solution to the underlying attitude stabilization problem by incorporating integral feedback action.

Theorem 1. Consider the system governed by Eqs. (3) and (1) with $\boldsymbol{d}(t) = 0$ for all $t \geq 0$. Assume that there exist scalar constants $k_i \geq 0$, $k_x \neq k_i/2$, and $k_p > k_i^2/2$. (All design parameters have been set to scalar values to permit ease in algebra and analysis. They can however be easily replaced by appropriate order matrices without sacrificing any theoretical assurances.) Let the feedback control torque $\boldsymbol{u}(t)$ be computed by

$$\begin{aligned} \boldsymbol{u} = &- \underbrace{[k_p + 2k_x(k_x - k_i)\beta_0]\boldsymbol{\beta}_v}_{\text{"Proportional Term"}} \\ &- \underbrace{k_i(k_x - k_i/2)[\beta_0 I_3 - S(\boldsymbol{\beta}_v)]\int_0^t \boldsymbol{\beta}_v(\tau)\,\mathrm{d}\tau}_{\text{"Integral Term"}} \\ &- \underbrace{(k_x - k_i/2)[S(\boldsymbol{\beta}_v) - \beta_0 I_3]\boldsymbol{x}}_{\text{"Filter Term"}} \end{aligned} \tag{7}$$

where the filter state $\boldsymbol{x}(t) \in \mathcal{R}^3$ is governed by the following first-order dynamics:

$$\dot{\boldsymbol{x}} = -\boldsymbol{x} + 2k_x\boldsymbol{\beta}_v + k_i \int_0^t \boldsymbol{\beta}_v(\tau), \qquad \text{any} \qquad \boldsymbol{x}(0) \in \mathcal{R}^3 \tag{8}$$

Then, all of the trajectories of the resulting closed-loop system are guaranteed to be globally uniformly bounded, and furthermore, they asymptotically converge to the set defined by Ω.

Before we proceed with the proof for theorem 1, the following remarks are in order:

1) The control scheme defined through Eq. (7) is rigorously independent of the angular velocity state $\boldsymbol{\omega}$. Whereas it involves computation of the filter state $\boldsymbol{x}$, one must observe that the filter state dynamics governed through Eq. (8) neither estimate the angular velocity nor approximate it through approximate numerical differentiation. Specifically, the nature of Eq. (8) suggests an asymptotically stable linear low-pass filter implementation that is forced by both the attitude variable $\boldsymbol{\beta}_v$ and its integral.

2) The filter construction is motivated from work in Refs. 3 and 4 with the addition of an integral term as a driving input in the filter dynamics. Another common approach to deal with the output feedback problem is highlighted in Refs. 11 and 12. This approach describes the construction of a high-pass filter to generate pseudovelocity signals from attitude measurements to be used in the feedback control law. For illustration sake, we present the scalar version of the filter here:

$$e_f = -ke + p, \qquad \dot{p} = -(k+1)p + (k^2+1)e \tag{9}$$

where $k > 0$ is a scalar, p is the filter state, and e is the attitude error (driving signal for the filter dynamics). As an extension of the filter proposed by Lizzeralde and Wen,[3] we propose an introduction of an integral term in the filter driving input, which will later enable us in the disturbance rejection process. The form of the filter is summarized as follows:

$$e_f = p, \qquad \dot{p} = -p + ke + k_i \int_0^t e\,\mathrm{d}\tau \tag{10}$$

The structure of the filter in Eq. (10) clearly shows that the output e_f in Eq. (10) is less noisy than that in Eq. (9), if the attitude measurements themselves are noisy. Further, unlike the output of the filter in Eq. (9), Eq. (10) do not estimate the time derivative of the attitude error.

3) Setting the integral feedback gain $k_i = 0$ within Eq. (7) recovers the well-known passivity-based angular velocity independent attitude control solution of Ref. 3. On the other hand, retaining a nonzero

value for k_i permits additional flexibility within the control law that can provide potential benefits in the presence of nonzero constant external disturbance torques. This is further elaborated in Sec. IV.

Proof of Theorem 1: The proof uses elements of Lyapunov stability theory and is organized as follows: we first propose a candidate Lyapunov function similar to the one used in Ref. 13, which is globally decrescent and radially unbounded in the states $\boldsymbol{\beta}_v$ and $\boldsymbol{\omega}$; then we prove that the time derivative of this Lyapunov function is negative semidefinite along trajectories generated by Eqs. (3), (1), and (8). Finally, we invoke uniform continuity type of arguments associated with Barbalat's lemma to show that the closed-loop system is globally asymptotically stable in the sense of theorem 1.

Now, consider the Lyapunov function candidate

$$V = \tfrac{1}{2}\boldsymbol{\omega}^T J \boldsymbol{\omega} + k_p\left[(\beta_0 - 1)^2 + \boldsymbol{\beta}_v^T \boldsymbol{\beta}_v\right] + \tfrac{1}{2}\dot{\boldsymbol{x}}^T\dot{\boldsymbol{x}} - k_i \boldsymbol{\beta}_v^T \dot{\boldsymbol{x}} \quad (11)$$

which can be shown to be positive definite and proper so long as k_p is selected according to the constraint $k_p > k_i^2/2$. Notice the presence of the cross term involving $\dot{\boldsymbol{x}}$ and the vector part of the Euler parameters $\boldsymbol{\beta}_v$. If the integral feedback gain k_i is set equal to zero, the cross term vanishes within the Lyapunov function candidate defined in Eq. (11), and one recovers the same choice of Lyapunov function employed within existing passivity-based attitude stabilization schemes.[3,5] Now, the time derivative of V taken along trajectories of the closed-loop system can be evaluated through laborious yet relatively straightforward algebra followed by application of the control torque Eq. (7) and can be stated as

$$\dot{V} = -\|\dot{\boldsymbol{x}} - k_i \boldsymbol{\beta}_v\|^2 \le 0 \quad (12)$$

Thus, $V(t)$, $\boldsymbol{\omega}(t)$, and $\dot{\boldsymbol{x}}(t)$ are all uniformly bounded. Recall that $\boldsymbol{\beta}_v$ is always bounded (unit norm constraint on the Euler parameter). Also, because $V \ge 0$ and $\dot{V} \le 0$, we have that $\lim_{t\to\infty} V(t) = V_\infty$ exists for some finite $V_\infty \in \mathcal{R}^+$. Hence, from Eq. (12) we have

$$\int_0^\infty \|\dot{\boldsymbol{x}} - k_i \boldsymbol{\beta}_v\|^2 \, \mathrm{d}t = V(0) - V_\infty$$

which implies that $\dot{\boldsymbol{x}} - k_i \boldsymbol{\beta}_v \in \mathcal{L}_2$. From Eq. (8), we have $\ddot{\boldsymbol{x}} = -\dot{\boldsymbol{x}} + 2k_x \dot{\boldsymbol{\beta}}_v + k_i \boldsymbol{\beta}_v$, which together with Eq. (3) implies $\ddot{\boldsymbol{x}} - k_i \dot{\boldsymbol{\beta}}_v \in \mathcal{L}_\infty$. Using Barbalat's lemma for $\dot{\boldsymbol{x}} - k_i \boldsymbol{\beta}_v \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\ddot{\boldsymbol{x}} - k_i \dot{\boldsymbol{\beta}}_v \in \mathcal{L}_\infty$, we can conclude that $\dot{\boldsymbol{x}}(t) - k_i \boldsymbol{\beta}_v \to 0$ as $t \to \infty$. Next, by virtue of the fact that the third time derivative of function

$$z(t) \doteq \boldsymbol{x}(t) - k_i \int_0^t \boldsymbol{\beta}_v(\tau)\, \mathrm{d}\tau$$

is uniformly bounded, we conclude uniform continuity for $\ddot{z}(t)$. Also, because, $\lim_{t\to\infty} \dot{z}(t) = 0$, we additionally have

$$\lim_{t\to\infty} \int_0^t \ddot{z}(\tau)\, \mathrm{d}\tau + \dot{z}(0) = 0$$

which when interpreted together with the uniform continuity of $\ddot{z}(t)$ permits the reapplication of the Barbalat's lemma (using the alternate statement of this lemma from Ref. 14) and finally leads to $\lim_{t\to\infty} \ddot{z}(t) = 0$, that is, $\lim_{t\to\infty}[\ddot{\boldsymbol{x}}(t) - k_i \dot{\boldsymbol{\beta}}_v(t)] = 0$. Notice though that $\ddot{z} = \ddot{\boldsymbol{x}} - k_i \dot{\boldsymbol{\beta}}_v = -(\dot{\boldsymbol{x}} - k_i \boldsymbol{\beta}_v) + 2(k_x - k_i/2)\dot{\boldsymbol{\beta}}_v$, we have $\lim_{t\to\infty} \dot{\boldsymbol{\beta}}_v(t) = 0$ whenever $\lim_{t\to\infty} \ddot{z}(t) = 0$ and $(k_x - k_i/2) \ne 0$.

Starting now with the unit norm constraint on the quaternion vector $\beta_0(t)^2 + \boldsymbol{\beta}_v^T(t)\boldsymbol{\beta}_v(t) = 1$ and differentiating both sides with respect to time, it follows that $\lim_{t\to\infty} \beta_0(t)\dot{\beta}_0(t) = 0$ after making use of $\lim_{t\to\infty} \dot{\boldsymbol{\beta}}_v(t) = 0$. Therefore we have the following cases: 1) $\lim_{t\to\infty} \beta_0(t) = 0$ and/or 2) $\lim_{t\to\infty} \dot{\beta}_0(t) = 0$. Let us initially assume that $\lim_{t\to\infty} \beta_0(t) = 0$, but $\lim_{t\to\infty} \dot{\beta}_0(t) \ne 0$. In this case, using boundedness of various signals on the right-hand sides of Eqs. (1) and (3), it is possible to conclude that $\ddot{\beta}_0(t)$ is bounded, thereby implying uniform continuity for $\dot{\beta}_0(t)$. Also

$$\lim_{t\to\infty} \beta_0(t) = 0 = \lim_{t\to\infty} \int_0^t \dot{\beta}_0(\tau)\, \mathrm{d}\tau$$

and using Barbalat's lemma, we obtain $\lim_{t\to\infty} \dot{\beta}_0(t) = 0$, which contradicts our assumption that $\lim_{t\to\infty} \dot{\beta}_0(t) \ne 0$. Therefore, irrespective of whether it is case 1) and/or case 2), it always follows that $\lim_{t\to\infty} \dot{\beta}_0(t) = 0$. However, at this point, it still remains to be shown as to what happens to $\beta_0(t)$ as $t \to \infty$.

We proceed by using the facts that $\dot{\boldsymbol{\beta}}_v(t) = 0$ and $\dot{\beta}_0(t) = 0$ as $t \to \infty$, which means that the left-hand side of Eq. (3) equals zero. Premultiplying both sides of Eq. (3) by the matrix $E^T(\boldsymbol{\beta})$ and using the identity stated in Eq. (5), it follows that $\lim_{t\to\infty} \boldsymbol{\omega}(t) = 0$. Next, Eq. (1) can be differentiated with time to show the fact that $\ddot{\boldsymbol{\omega}} \in \mathcal{L}_\infty$, which implies that $\dot{\boldsymbol{\omega}}(t)$ is uniformly continuous, or by applying Barbalat's lemma one more time, we have $\dot{\boldsymbol{\omega}}(t) \to 0$ as $t \to \infty$. This last result together with the control law given in Eq. (7) can be used in Eq. (1) to demonstrate that $-(k_p - k_i k_x \beta_0)\boldsymbol{\beta}_v(t) - (k_x - k_i/2)[\beta_0 I_3 - S(\boldsymbol{\beta}_v)]\dot{\boldsymbol{x}} \to 0$ as $t \to \infty$. Now, premultiplying both sides with $\boldsymbol{\beta}_v^T(t)$, we obtain $\lim_{t\to\infty}(k_p - \beta_0 k_i^2/2)\|\boldsymbol{\beta}_v(t)\|^2 = 0$. Because the scalar k_p is selected according to $k_p > k_i^2/2$ and $|\beta_0| \le 1$ (unit-norm constraint), it therefore follows that $\lim_{t\to\infty} \boldsymbol{\beta}_v(t) = 0$. Once again using the unit-norm constraint on the Euler parameter vector $\boldsymbol{\beta}(t)$, it follows that $\lim_{t\to\infty} \beta_0(t) \ne 0$ [more precisely, $\lim_{t\to\infty} \beta_0(t) = \pm 1$]. Thus, we are able to show that

$$\lim_{t\to\infty} [\boldsymbol{\beta}_v(t), \boldsymbol{\omega}(t)] = 0$$

thereby completing the proof of achieving the stated attitude stabilization objective.

It can further be shown that a direct linearization of the closed-loop system after substituting the control law yields a type-I system for the transfer function between the disturbance input and the attitude vector components, thereby illustrating the disturbance rejection properties of the control law derived in Eq. (7).

IV. Numerical Simulation Results

As a test for the new PI control law derived earlier, we consider a representative problem of regulating the orientation of a rigid body to the origin. The inertia matrix is specified by

$$I = \begin{bmatrix} 3 & 0.1 & 0.2 \\ 0.1 & 4 & -0.3 \\ 0.2 & -0.3 & 5 \end{bmatrix}$$

At time $t = 0$, the orientation of the rigid body is such that the initial attitude described is $\boldsymbol{\beta}(0) = [0,\ 1/2\sqrt{2},\ -1/2\sqrt{2},\ -\sqrt{3/2}]^T$ with a zero initial body angular velocity. Further, we impose a constant external disturbance torque specified by $\boldsymbol{d}(t) = [0.2,\ 0.1,\ -0.1]^T$. The initial condition for the filter state is chosen $\boldsymbol{x}(0) = 0$, and the following choices are made for the various design parameters: $k_x = 1$, $k_p = 1 + k_i^2/2$.

In the first case, we chose the integral feedback gain $k_i = 0$ amounting to the standard passivity-based attitude control algorithm. The resulting trajectories are plotted in Fig. 1. Time histories of the vector part of the quaternion $\boldsymbol{\beta}_v(t)$, the angular velocity $\boldsymbol{\omega}(t)$, and the control torques $\boldsymbol{u}(t)$ are shown using $k_i = 0$. Also shown in Fig. 1 is the norm of the attitude error $\|\boldsymbol{\beta}_v(t)\|$. All of the closed-loop trajectories can be seen to be bounded. The control torque "learns" and cancels the constant disturbance value so that the angular velocity is regulated to zero. However, as expected, eliminating the integral feedback induces a nonzero steady-state error for the orientation vector.

For our next set of simulations, we use a nonzero integral feedback gain, that is, $k_i = 0.3$ within our control law described in Eq. (7). All other design variables and initial conditions are kept unchanged from the preceding simulation except for a slightly larger k_p value as dictated by the condition from theorem 1, that is, $k_p = 1 + k_i^2/2 > k_i^2/2$. The resulting closed-loop trajectories are shown in Fig. 2. We observe from these simulations that although

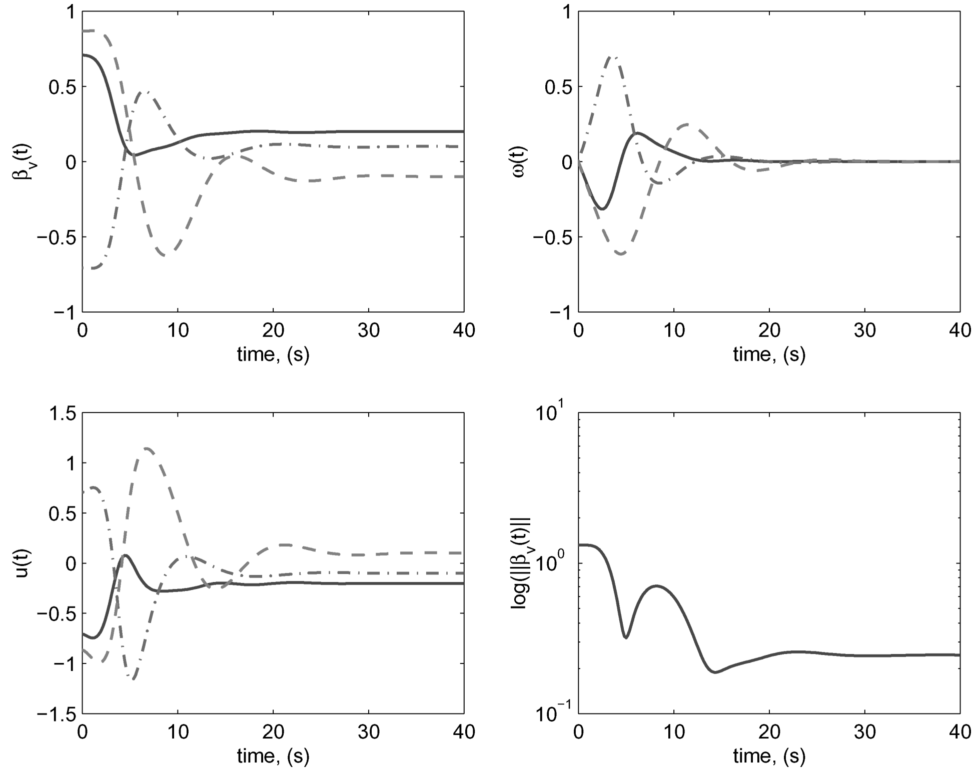

Fig. 1 Closed-loop trajectories resulting from standard passivity-based (nonintegral feedback) controller.

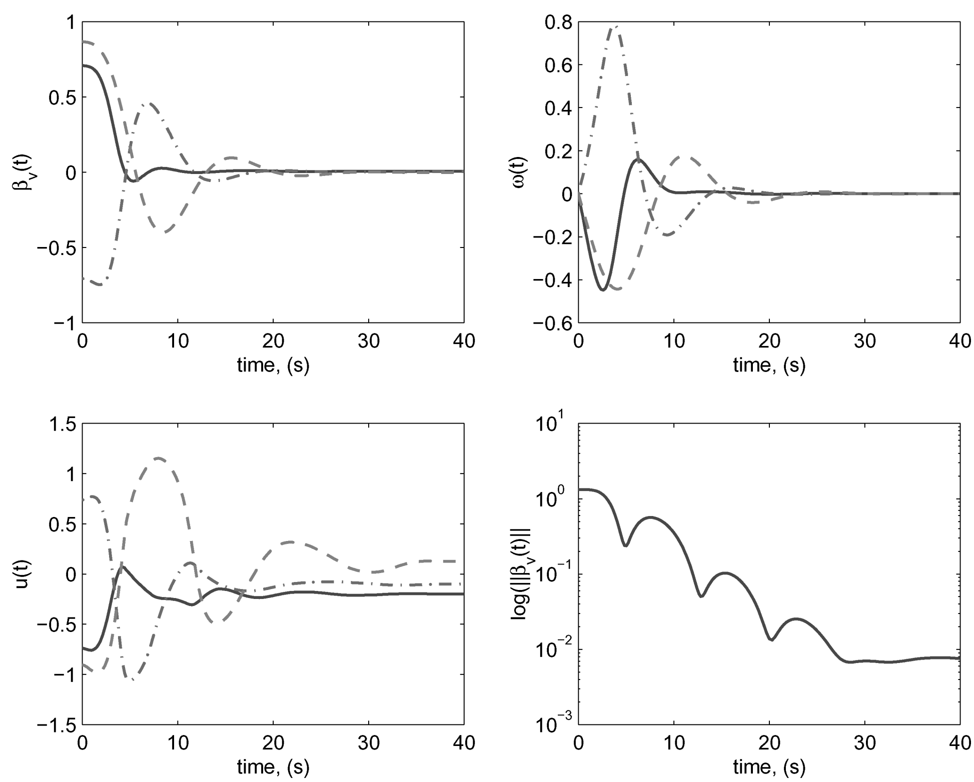

Fig. 2 Closed-loop trajectories resulting from PI feedback controller.

the controller is once again able to reject the constant external disturbance torque the steady-state error in the attitude stabilizes to a much smaller equilibrium value when compared to the $k_i = 0$ case. (Norm of the attitude vector is plotted in both Figs. 1 and 2 to enable comparison.) On the other hand, the speed and quality of the closed-loop response is nearly identical with and without integral feedback implying that the slowdown in the performance as a result of the integral feedback action is virtually undetectable on the scale of these plots. In this fashion, we have verified through these simulations that the steady-state values of the attitude errors can be significantly improved as a result of the integral feedback action without paying much penalty in terms of either increases in control torque magnitudes or reduction in control speeds. As opposed to the classical passivity-based control torque solution provided by setting $k_i = 0$ within Eq. (7) of theorem 1, the feature of having an additional degree of freedom afforded through tuning the integral feedback gain parameter k_i can be construed as a distinct advantage of the new proportional-integral control law derived in this Note.

V. Conclusions

The main contribution of Note is the design of a new class of angular velocity-free attitude stabilization controllers that can be viewed as generalizations of the existing passivity-based control algorithms in the sense of providing an additional provision for integral feedback action. Through a rigorous Lyapunov analysis, we have demonstrated global asymptotic stability for the full state of the rigid-body rotational motion when the external disturbances are absent. The disturbance rejection aspect is illustrated using a linearized setting of the problem. The controller structure includes a low-pass linear filter state that is derived without explicit differentiation of attitude to synthesize angular velocity-like signals. Numerical implementation of these new results provide the assurance of significantly improved steady-state attitude error convergence in the presence of constant unknown external disturbances as a result of integral feedback action. Whereas in this study we have adopted the once-redundant Euler parameter representation for the attitude kinematics, the main results presented here can be readily replicated in terms of other kinematic representations derived from the Euler principal rotation theorem such as the Gibbs vector and the modified Rodrigues parameters.

Acknowledgments

These results are based in part upon work supported by the Texas Advanced Research Program under Grant 003658-0081-2001.

References

[1]Junkins, J. L., and Turner, J. D., *Optimal Spacecraft Rotational Maneuvers*, Elsevier Science, Amsterdam, 1986, Chap. 8.

[2]Schaub, H., and Junkins, J. L., *Analytical Mechanics of Space Systems*, AIAA Education Series, AIAA, Reston, VA, 2003, Chap. 8.

[3]Lizarralde, F., and Wen, J. T., "Attitude Control Without Angular Velocity Measurement: A Passivity Approach," *IEEE Transactions on Automatic Control,* Vol. 41, No. 3, 1996, pp. 468–472.

[4]Tsiotras, P., "Further Passivity Results for the Attitude Control Problem," *IEEE Transactions on Automatic Control*, Vol. 43, No. 11, 1998, pp. 1597–1600.

[5]Akella, M. R., "Rigid Body Attitude Tracking Without Angular Velocity Feedback," *Systems and Control Letters*, Vol. 42, No. 4, 2001, pp. 321–326.

[6]Akella, M. R., and Kotamraju, G. R., "Rigid-Body Output Feedback Attitude Control Subject to Actuator and Rate Saturations," *14th US Congress on Theoretical and Applied Mechanics*, U.S. National Committee on Theoretical and Applied Mechanics, 2002.

[7]Subbarao, K., "The Attitude Control Problem, Re-Visited," Advances in the Astronautical Sciences, The John L. Junkins Astrodynamics Symposium, Vol. 115, Univelt, San Diego, CA, 2003, pp. 95–110.

[8]Paielli, R. A., and Bach, R. E., "Attitude Control with Realization of Linear Error Dynamics," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 1, 1993, pp. 182–189.

[9]Shuster, M. D., "A Survey of Attitude Representations," *Journal of the Astronautical Sciences*, Vol. 41, No. 4, 1993, pp. 439–517.

[10]Stanley, W. S., "Quaternion from Rotation Matrix," *Journal of Guidance and Control*, Vol. I, No. 3, 1978, pp. 223, 224.

[11]Burg, D., Dawson, J. H., and de Queiroz, M. S., "An Adaptive Partial State Feedback Controller For RLED Robot Manipulators," *IEEE Transactions of Automatic Control*, Vol. 41, No. 7, 1996, pp. 1024–1031.

[12]Wong, H., de Querioz, M. S., and Kapila, V., "Adaptive Tracking Control Using Synthesized Velocity From Attitude Measurements," *Automatica*, Vol. 37, No. 6, 2001, pp. 947–953.

[13]Ortega, R., Loria, A., Nicklasson, P. J., and Ramirez, H., *Passivity-Based Control of Euler-Lagrange Systems*, Springer-Verlag, New York, 1998.

[14]Ioannou, P. A., and Sun, J., *Stable and Robust Adaptive Control*, Prentice–Hall, Upper Saddle River, NJ, 1995, pp. 85–134, Chaps. 2–4.

Lyapunov-Based Nonlinear Missile Guidance

N. Léchevin*
Université du Québec à Trois-Rivières, Trois-Rivières, Quebec G9A 5H7, Canada
and
C. A. Rabbath†
Defence Research and Development Canada–Valcartier, Val-Belair, Quebec G3J 1X5, Canada

I. Introduction

PROPORTIONAL navigation guidance (PNG) has been widely used for decades because of its implementation simplicity and its effectiveness in guiding missiles to intercept nonmaneuvering targets.[1] Recently, new guidance laws have been proposed, based mainly on optimal control theory,[2,3] differential game theory,[4] the geometric approach,[5] and variable structure control[6] to meet the challenges posed by ever more agile targets. Improvements to PNG have also been proposed. For instance, neoclassical guidance[7,8] requires line-of-sight (LOS) rate measurements only to warrant zero-miss distance by rendering the kinematics-PNG-seeker-missile dynamic loop positive real. Novel guidance laws are often obtained in the context of linear systems theory. The argument for such linear synthesis is that small-angle engagement justifies linearization around an operating point and, in particular, around a null LOS rate. However, for the pursuit of highly maneuverable targets and to satisfy demanding precision guidance requirements, synthesis of guidance laws considering the nonlinear missile–target relative kinematics is intuitively expected to provide performance superior to that of a linear design relying on approximations. Nonlinear guidance laws providing performance improved over that of classical PNG have recently been proposed.[9,10] In this context, an issue arising with the selection of the Lyapunov function candidate relates to the guidance law potentially depending on terms such as $1/\cos(\lambda)$, where λ is the LOS angle.[10] The commanded acceleration may then become prohibitively large around LOS angles close to $\pm\pi/2$ rad. In this Note, we propose a quadratic Lyapunov function candidate resulting in guidance laws that are free of singularities, such as $\lambda = \pm\pi/2$ rad; and that provide reduced miss distances when compared to PNG. The cornerstone of the proposed approach lies in the particular selection of the state-space variables used in the Lyapunov-based synthesis, which are trigonometric functions of the LOS and LOS rate. Provided certain conditions are met, the proposed approach warrants uniform ultimate boundedness of the missile–target system state, namely LOS and LOS rate, for the case of highly maneuvering targets. For nonmaneuvering targets, asymptotic stability is demonstrated. Numerical simulations show the effectiveness of the proposed nonlinear guidance laws.

II. Mathematical Preliminaries

A. Missile–Target Kinematics

A two-dimensional engagement can be studied by assuming that the lateral and longitudinal planes are decoupled.[8] The engagement

Received 27 February 2004; revision received 15 June 2004; accepted for publication 6 July 2004.

*Adjunct Professor, Department of Electrical Engineering; Nicolas_Lechevin@uqtr.ca.

†Defence Scientist, 2459 Pie-XI N.; camille-alain.rabbath@drdc-rddc.gc.ca.